

DISCRETE-SIZING OPTIMAL DESIGN OF SCISSOR-LINK FOLDABLE STRUCTURES USING GENETIC ALGORITHM

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ABSTRACT

In this article the genetic algorithm is employed to optimize scissor-link foldable structures. The advantage of using GA lies in the fact that the discrete spaces can be optimized without any complexity. Here displacement method is used for analysis with uniplet elements.

Keywords: optimization, scissor-link foldable structures, design, uniplet, genetic algorithm

1. INTRODUCTION

Optimization of an engineering design is an improvement of a proposed design that results in the best properties for minimum cost. Optimization can be categorized as sizing optimization, shape optimization, topology optimization and layout optimization. Classical optimization is done manually with algebra, calculus, and calculus of variations. Many design problems are too complex to be handled with purely algebraic method. Evolutionary methods are other optimization methods that employ neural networks, simulated annealing or genetic algorithm.

The need for mobile, re-usable structures that are characterized by fast and easy erection existed for a long time. Such structures found application in the temporary construction industry. They are also used for recreational purposes, and providing solutions for quick sheltering after natural disasters. In recent years a new exciting area for applications is offered by the aerospace industry. The first such structure has been designed and constructed by Pinero [1]. Substantial contribution to the general understanding of geometric and kinematic behavior of scissor-link structures is due to Escrig [2], and Escrig and Valcarcel [3]. Further studies have been made by Ziegler [4], Derus [5], Nodskov [6], Gantes et al [7], Rosenfeld et al [8], Shan [9], and Kaveh and Davaran [10], covering various aspects of foldable structures.

In this article sizing optimization of scissor-link foldable structures employing Genetic algorithm is has been studied. In the process of optimal design, analysis should be performed several times. Here, by using the stiffness matrix of uniplet, analysis is simplified and the efficiency is increased.

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2. METHOD OF ANALYSIS

Efficient methods of analysis for foldable structures are developed by Shan [9], and Kaveh and Davaran [10]. Let the member of Figure 1 represents a typical uniplet from a scissor-link structure. The force-displacement relationship for a uniplet can be written as:

$$\mathbf{p}_u = \mathbf{k}_u \cdot \mathbf{d}_u \quad (1)$$

In this equation \mathbf{p}_u and \mathbf{d}_u are the force and displacement vectors of uniplet as

$$\begin{aligned} \mathbf{p}_u &= \{p_{ix}, p_{iy}, p_{iz} \mid p_{jx}, p_{jy}, p_{jz} \mid p_{kx}, p_{ky}, p_{kz}\} \\ \mathbf{d}_u &= \{d_{ix}, d_{iy}, d_{iz} \mid d_{jx}, d_{jy}, d_{jz} \mid d_{kx}, d_{ky}, d_{kz}\} \end{aligned} \quad (2)$$

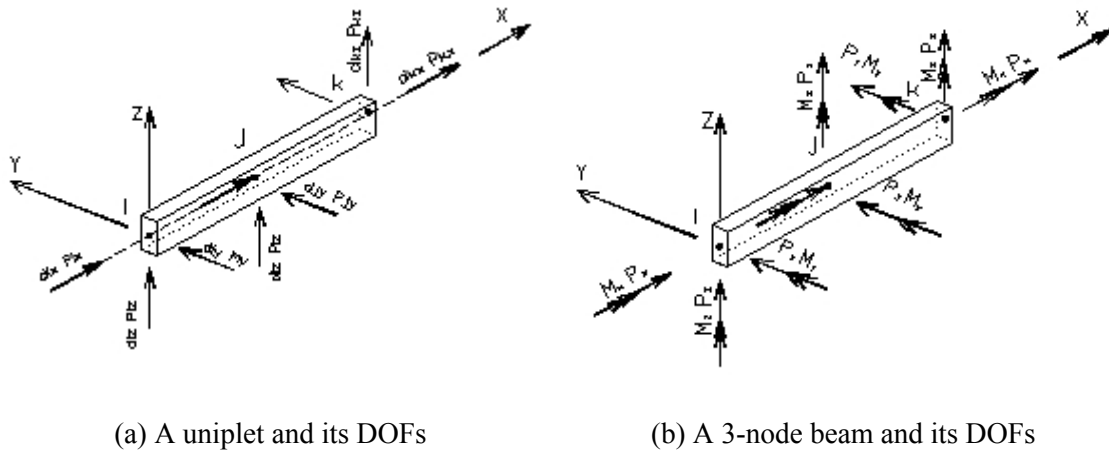


Figure 1. A uniplet and a 3-node beam.

Comparison between the uniplet and the 3-node beam shows that a uniplet may be considered as a 3-node beam without any moment applied at the joints and it has no torsional deformation. Thus the relation may symbolically be written as:

$$\begin{Bmatrix} \mathbf{p}_u \\ \mathbf{0} \end{Bmatrix} = \begin{bmatrix} \mathbf{k}_{11} & \mathbf{k}_{12} \\ \mathbf{k}_{21} & \mathbf{k}_{22} \end{bmatrix} \begin{Bmatrix} \mathbf{d}_u \\ \boldsymbol{\theta} \end{Bmatrix}, \quad (3)$$

where \mathbf{k}_{11} , \mathbf{k}_{12} , \mathbf{k}_{21} , \mathbf{k}_{22} , \mathbf{p}_u , \mathbf{d}_u and $\boldsymbol{\theta}$ are submatrices and subvectors. Here \mathbf{p}_u and \mathbf{d}_u are identical to the member force and displacement vectors for the uniplet. Expanding eqn (3) results in

$$\mathbf{k}_{21}\mathbf{d}_u + \mathbf{k}_{22}\boldsymbol{\theta} = \mathbf{0}, \quad (4)$$

$$\mathbf{p}_u = \mathbf{k}_{11}\mathbf{d}_u + \mathbf{k}_{12}\boldsymbol{\theta} . \quad (5)$$

Combining eqn (4) and eqn (5) leads to

$$\mathbf{p}_u = \mathbf{k}_{11}\mathbf{d}_u - \mathbf{k}_{12}\mathbf{k}_{22}^{-1}\mathbf{k}_{21}\mathbf{d}_u . \quad (6)$$

Comparison of eqns (1) and (6) leads to

$$\mathbf{k}_u = \mathbf{k}_{11} - \mathbf{k}_{12}\mathbf{k}_{22}^{-1}\mathbf{k}_{21} . \quad (7)$$

This gives rise to the stiffness matrix for the uniplet [9,10].

3. STRUCTURAL OPTIMIZATION USING GENETIC ALGORITHMS

Goldberg is one of the pioneers in developing the genetic algorithm [11]. In structural engineering, Goldberg and Samtani [12], Rajeev and Krishnamoorthy [13], Jenkins [14], Lin and Hajela [15], Adeli and Chung [16], Saka and Kameshki [17], Kaveh and Kalatjari [18-20] show that genetic algorithm is a powerful tool for the optimization of structures.

Optimization by GA consists of three steps:

In the first step random creation of a primary design population is performed. Each design is called an individual and the number of individuals is known as the pop-size. Each individual consists of a string of characters. Each character is usually a random binary number, and the number of characters or bits shows the length of each individual. Each string consists of some substrings and each substring represents a design variable. The number of substring is equal to the number of design variables involved in an optimization. The individual and character in GA terminology are the same as the chromosome and gene in natural genetics, respectively.

In the second step, after producing a primary population, by decoding the strings, real values of design variables are evaluated. Then the magnitude of the objective function, member stresses, joint displacement and the magnitudes of constraint violations corresponding to structural response are obtained. In the third step, a penalty function is defined representing the violation of constraints, that combined with the objective function, leads to the modified objective function.

In this manner a constrained optimization is changed to an unconstrained optimization. Defining a fitness function, for the modified objective function corresponding to each individual, a fitness value is obtained. Using the selection process inspired by the natural evolution of living organisms, individuals with high fitness are selected for reproduction. The latter is carried out using the roulette wheel rule. Crossover and mutation is then performed and a new population is created. The crossover operator performs mating between two arbitrary individuals to produce offspring. The number of such operations depends on the crossover rate. The mutation operator randomly changes some 0 characters to 1 and vice versa according to a predicted mutation rate. Both operators are used to produce diversity in the search space to increase the chance of obtaining a global optimum.

3.1 GA based discrete-sizing scissor-link foldable structure optimization

Discrete optimal design of scissor-link foldable structures using GA can be formulated as

$$\text{Find } X = [X_1, X_2, \dots, X_{ns}]^T ; X_s \in S; s = 1, \dots, ns \quad (8)$$

$$\text{to minimize } \phi(x) = f(x) + f_{\text{penalty}} \quad (9)$$

Subjected to the following constraints:

For combined bending and compression:

$$g_{i1}(x) = 1 - \frac{f_a}{F_a} \frac{c_m f_b}{(1 - \frac{f_a}{F_e}) F_b} \geq 0.0 ; i = 1, \dots, ne \quad (10)$$

or

$$g_{i1}(x) = 1 - \frac{c_m f_b}{(1 - \frac{f_a}{F_e}) F_b} \geq 0.0 ; i = 1, \dots, ne \quad (11)$$

$$g_{i2}(x) = 1 - \frac{f_a}{\frac{F_y}{\gamma}} - \frac{f_b}{F_b} \geq 0.0 ; i = 1, \dots, ne \quad (12)$$

For combined bending and tension:

$$g_i(x) = 1 - \frac{f_a}{\frac{F_y}{\gamma}} - \frac{f_b}{F_b} \geq 0.0 ; i = 1, \dots, ne \quad (13)$$

$$g_{i2}(x) = 1 - \frac{f_a}{\frac{F_y}{\gamma}} - \frac{f_b}{F_b} \geq 0.0 ; i = 1, \dots, ne \quad (14)$$

$$g_j(x) = 1 - \left| \frac{\Delta_j}{\Delta_a} \right| \geq 0.0 ; j = 1, \dots, ndof \quad (15)$$

Where X is the vector containing the design variables, X_s is the cross-sectional area for the s th group belonging to the variable profile list S , ns is the number of design variables or number of member grouping, $f(x)$ is the objective function which is usually taken as the weight or volume of the structure, f_{penalty} is the penalty function which results from the violations of the constraints corresponding to the response of the structure, $\phi(x)$ is the modified objective function, f_a is maximal compressive stress due to axial loading, F_a is allowable stress for axial load alone, f_b is

maximal bending stress at furthest fiber of the cross section, F_b is allowable stress for bending

$$\text{alone, } F'_e = \frac{F_e}{\gamma}, F_e = \frac{\pi^2 E}{\left(\frac{kl}{r}\right)^2}, \gamma = \frac{5}{3} + \frac{3}{8} \frac{\left(\frac{kl}{r}\right)}{c_c} - \frac{1}{8} \left(\frac{kl}{r}\right)^3, c_c = \pi \sqrt{\frac{2E}{F_y}}, k \text{ is effective length}$$

coefficient that is taken as unit, l is member length, r is radius of gyration, c_m is a coefficient that is taken as unit, Δ_j is the displacement at that degree of freedom, Δ_a is the displacement permitted by the code of practice. The objective function in the form of the weight of the structure is as follows:

$$f(x) = \sum_{i=1}^{ne} X_i l_i \rho_i \quad (16)$$

3.2 Penalty function

Different a penalty function are used in the literature. One of the earliest is due to Rajeev and Krishnamoorthy [13] as follows:

$$f_{\text{penalty}} = f(x) k_p c ; c = \sum_{q=1}^{nc} \max[0, g_q(x)] \quad (17)$$

In this relation, $nc = ne + ndof + mc$ represents the number of evaluated constraints for each individual design. The constant k_p is taken 3.6 and 10 for two different types of penalty function used in this study.

3.3 Fitness function

The fitness function is also defined in different forms. Rajeev and Krishnamoorthy [13] have suggested the following function that is used in this paper:

$$F_i = [\phi(x)_{\max} + \phi(x)_{\min}] - \phi(x) \quad (18)$$

In this equation, F_i and $\phi(x)$ are the fitness and the value of modified objective of the i th individual, $\phi(x)_{\max}$ and $\phi(x)_{\min}$ are the maximum and minimum values of the modified objective function in the population, respectively.

3.4 Decoding an individual in discrete-sizing optimization

As mentioned before, each individual consists of substrings represented by encoded design variables. For discrete-size optimization of scissor-link foldable structures, design variables are the cross-sections of grouped members and are selected from an available profile list, i.e. chosen from the set $S = \{s_1, s_2, \dots, s_{ns}\}$. If the length of the i th substrings, l_i , represent the number of bits or characters, and at the stage of encoding, random binary numbers are associated with them, in the process of decoding, 2^{l_i} values can be addressed of which the smallest is the integer 0 and the largest is the integer $2^{l_i} - 1$. One can use the following general relations in order to

correspond the integer random value IR_i related to the i th substring to the section number IS_i in the profile list S .

$$\begin{aligned} IS_i &= IND_i + IR_i \quad ; \quad i = 1, 2, \dots, nd \\ 1 &\leq IS_i \leq ns \\ 1 &\leq IND_i \leq ns - 2^{l_i} + 1 \\ 0 &\leq IR_i \leq 2^{l_i} - 1 \end{aligned} \tag{19}$$

In the above relation nd represents the number of substrings which is the same as the number of design variables or the number sections in the list S . The random number IS_i is the index which recognizes the section area $A_i = IS_i$ for the i th substring in S . If IND_i is taken as unity, the ns is preferred to be such that $2^{l_i} = ns$ yields an integer for l_i .

3.5 Termination criterion

For discrete optimization problems, the procedure can be terminated when one of the following heuristic criteria is satisfied [21]:

- I. When the best value of the objective function remains unchanged in the last $\frac{4nN_s}{N_{ns}}$ generations.
- II. When the mean value of the objective function from all parent vectors in the last $\frac{4nN_s}{N_{ns}}$ generations has not improved by less than a given value, say 0.0001
- III. When the relative difference between the best objective function value and the mean of the values of all the objective functions of the parent vectors in the current generation is less than a specified value, say 0.0001
- IV. When the ratio N_{s0}/N_s has reached a given value between 0.5 and 0.8 where N_{s0} is the number of parent vectors in current generation with the best objective function value, N_{ns} is the number of offspring, N_s is the number of population.

In this paper, the criterion III is used for termination of the GA operations.

4. NUMERICAL EXAMPLES

For justification of the developed algorithm first a classical example of space trusses is studied. Then two examples of foldable structures optimized using the present algorithm.

Example 1 (a 25-bar space truss): A 25-bar truss shown in Figure 3 was considered to be optimized. In table I, the members are grouped in eight sets. Table 2 contains set of discrete cross-sections with 32 elements. Table III contains all the design data. In this example, the effect

of buckling in is not included.

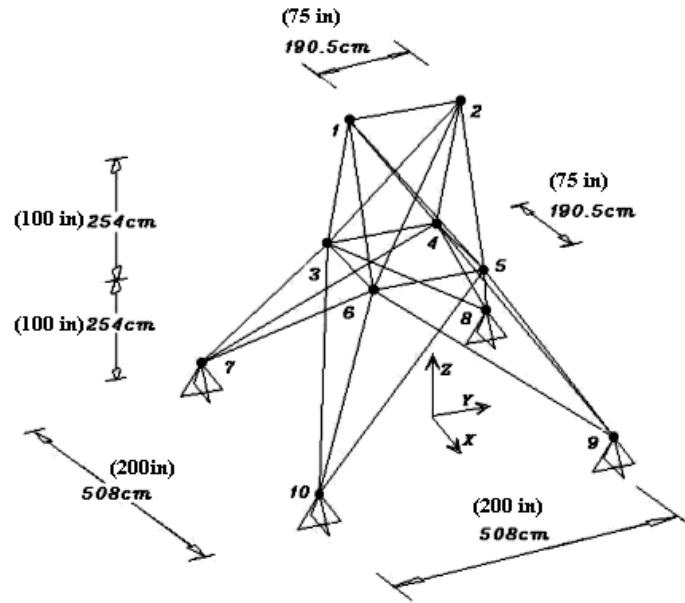


Figure 3. A 25-bar space truss

Table 1: Details for grouping of the 25-bar space truss

Group numbers	Design variables	Nodal numbers of the members	Member numbering
1	X_1	1-2	1
2	X_2	1-4,2-3,1-5,2-6	2,3,4,5
3	X_3	2-5,2-4,1-3,1-6	6,7,8,9
4	X_4	3-6,4-5	10,11
5	X_5	3-4,5-6	12,13
6	X_6	3-10,6-7,4-9,5-8	14,15,16,17
7	X_7	3-8,4-7,6-9,5-10	18,19,20,21
8	X_8	3-7,4-8,5-9,6-10	22,23,24,25

Table 2: The discrete cross-section set

$S = \{0.645 \times I (I=1,2,\dots,26), 18.064, 19.355, 20.645, 21.935, 22.538, 23.5\} (\text{cm}^2)$
$S = \{0.1 \times I (I=1,2,\dots,26), 2.8, 3.0, 3.2, 3.4, 3.5, 3.65\} (\text{in}^2)$

Table 3: Design data

Constraint data	Displacement constraints: In the direction of X and Y axis $\Delta_j \leq 0.889\text{cm (0.35 in)}$ Stress constraints: $\sigma_i \leq 275.6 \text{ Mpa (40Ksi)}$ $i = 1, 2, \dots, 25$			
Loading data	Nodal number	$P_x (kN)$	$P_y (kN)$	$P_z (kN)$
	1	4.545	-44.537	-44.537
	2	0	-44.537	-44.537
	3	2.227	0	0
	6	2.672	0	0
Material properties	Modulus of elasticity $E = 6.895 \times 10^4 \text{ Mpa}$ (10^4 ksi)			
	Weight density of the material $\rho = 0.0272 \text{ N/cm}^3$ (0.1 lb/in^3)			

Table 4 contains the optimal design of the 25-bar truss with this algorithm. Figure 4 shows the history of optimization for generations. In this example, the number of generations is taken as 50, the population size is chosen as 100, the mutation rate is 0.15, and the

constant k_p for penalty function is taken 3. 6.

Table 4: Comparison of the results for the 25-bar truss

Method	Weight N (lb)	Design variables cm^2							
		X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
Rajeev et al. [13]	2431.79 (546.01)	0.645	11.613	14.839	1.29	0.645	5.161	11.613	19.355
Zhu [19]	2457.44 (562.9)	0.645	12.258	16.774	0.645	0.645	5.161	13.548	16.774
Erbatur et al [20]	2199.26 (493.8)	0.645	7.742	20.645	0.645	7.097	5.806	2.581	21.935
Kaveh Kalatjari [17]	2138.82 (480.23)	0.645	0.645	22.581	0.645	12.903	6.452	0.645	25.806
Present work	2210.87 (506.44)	0.645	3.225	19.355	0.645	10.965	7.74	1.935	19.355

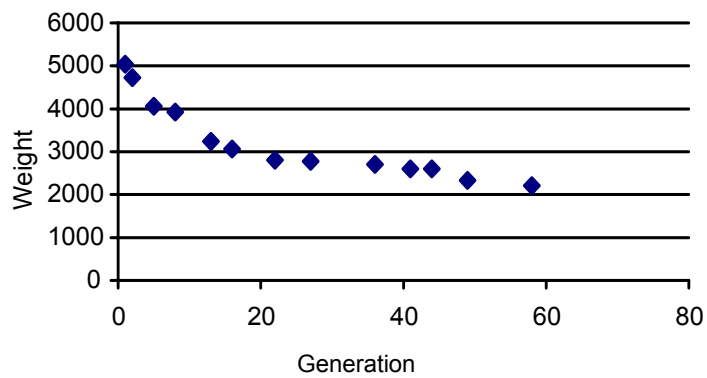


Figure 4. The optimization history of 25-bar truss

Example 2 (a 32-uniplet foldable barrel): The minimal weight design of the 32-uniplet foldable barrel vault type of structure, as shown in Figure 5, is performed. In figure 6,7 the geometrical properties are considered. Table 5 contains the numbering of nodes and member ordering. In table 6 the set of cross-sections are grouped in 16 elements. Table 7 contains all the design data.

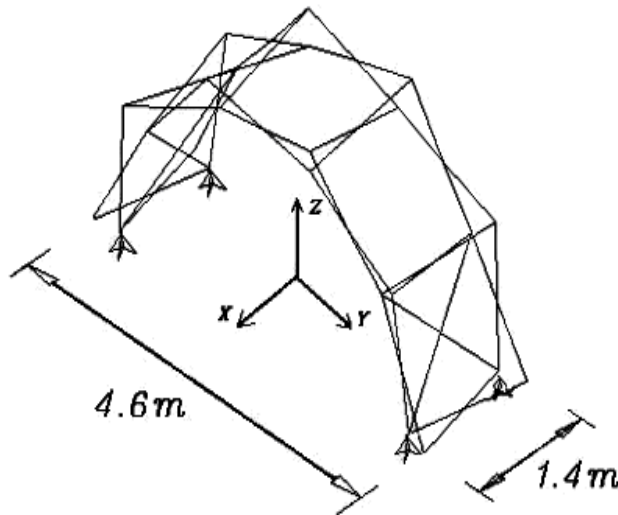


Figure 5. A 32-uniplet foldable barrel

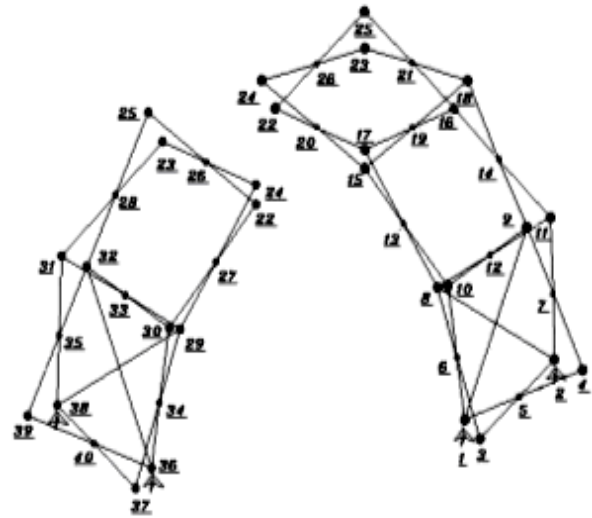


Figure 6. Numbering of nodes

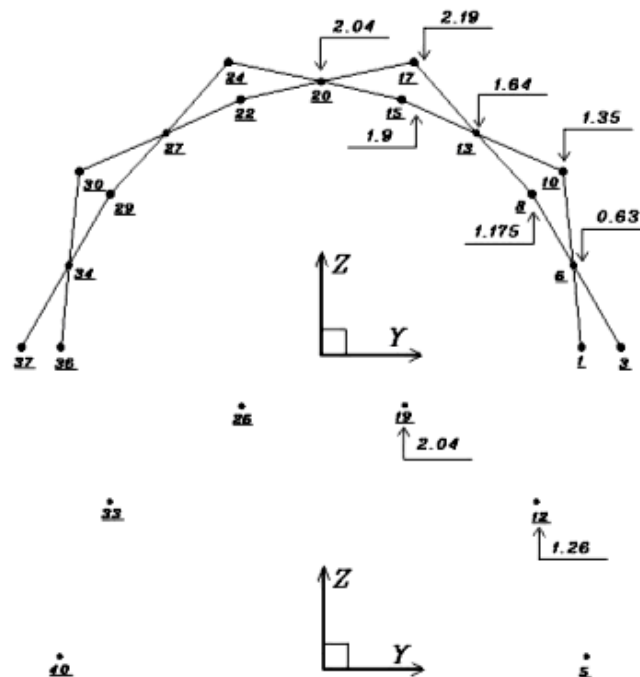


Figure 7. Elevation of the nodes

Table 5: Numbering of nodes and member ordering

Group numbers	Design variables	Nodal numbers of 3-node members	Member numbering
1	X_1	1-6-10, 2-7-11 30-34-36, 31-35-38	1, 2 3, 4
2	X_2	3-6-8, 4-7-9 8-13-17, 10-13-15 11-14-16, 9-14-18 29-34-37, 32-35-39 24-27-29, 30-27-22 25-28-32, 31-28-23	5, 6 7, 8 9, 10 11, 12 13, 14 15, 16
3	X_3	9-10-12, 11-12-18 1-5-4, 3-5-2 30-33-32, 29-33-31 37-40-38, 36-40-39	17, 18 19, 20 21, 22 23, 24
4	X_4	17-20-22, 15-20-24 18-21-23, 16-21-25	25, 26 27, 28
5	X_5	15-19-18, 17-19-16 22-26-25, 24-26-23	29, 30 31, 32

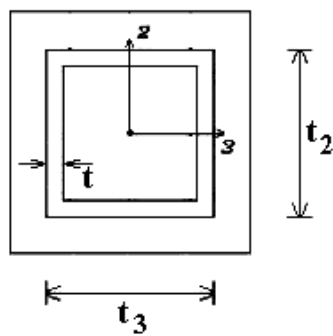


Figure 8. Element section

Table 6: The discrete cross-section set

$$S = \left\{ \begin{array}{l} 4^{\text{cm}} \times 3^{\text{cm}} \times I, 3^{\text{cm}} \times 3^{\text{cm}} \times I, 3^{\text{cm}} \times 2^{\text{cm}} \times I \\ 2^{\text{cm}} \times 2^{\text{cm}} \times I, 2^{\text{cm}} \times 1^{\text{cm}} \times I, 1^{\text{cm}} \times 1^{\text{cm}} \times 0.2^{\text{cm}} \end{array} \right\}, I = 0.2, 0.3, 0.4(\text{mm})$$

Table 7: Design data

Constraint data	Displacement constraints: In all direction $\Delta_j \leq 2\text{cm (0.79 in)}$ Stress constraints: Relations 10,11,12,13,14								
	Nodal#	10	11	17	18	22	23	24	25
Loading data	$-P_z(\text{kN})$	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95
Material properties	Modulus of elasticity $E = 2.06 \times 10^5 \text{Mpa } (3 \times 10^4 \text{ksi})$ Weight density of the material $\rho = 7.4 \times 10^4 \text{ N/m}^3 (0.27 \text{ lb/in}^3)$ Yield stress $f_y = 2.35 \times 10^2 \text{ Mpa}(34.08 \text{ksi})$								

In table 8 the results of optimal design are depicted. Figure 9 shows the history of optimization for generations. In this example, the number of generation is 50, the population size is chosen as 100, the mutation rate is 0.15, and the constant k_p for penalty function is taken as 10.

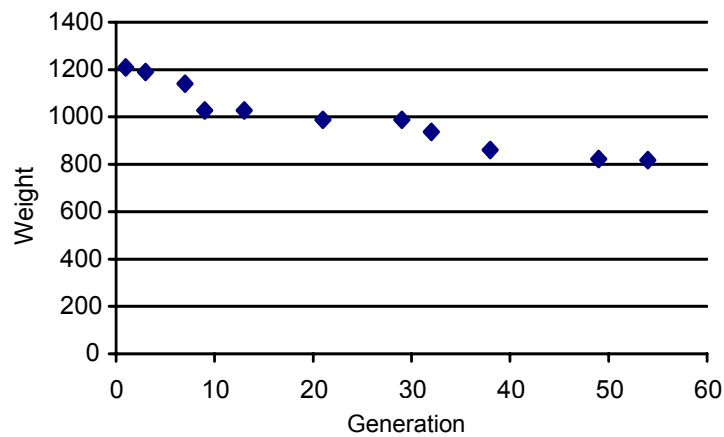


Figure 9. The history of optimization for the 32-uniplete foldable barrel vault

Table 8: Results for the 32-uniplet foldable barrel vault

Weight N (lb)	Design variables cm^2				
	X_1	X_2	X_3	X_4	X_5
817.38 (183.7)	4.96	2.56	2.64	1.29	0.64
Sec. #	16	7	9	8	1

Example 3: a 80-uniplet foldable dome optimal design of a 80-uniplet foldable dome, as shown in Figure 10 is considered. In Figures 11,12,13, and 14, the geometrical properties are shown. Table 9 contains the numbering of nodes and the member ordering. In table 10 the set of cross-sections are grouped in 16 elements. Table 11 contains all the design data.

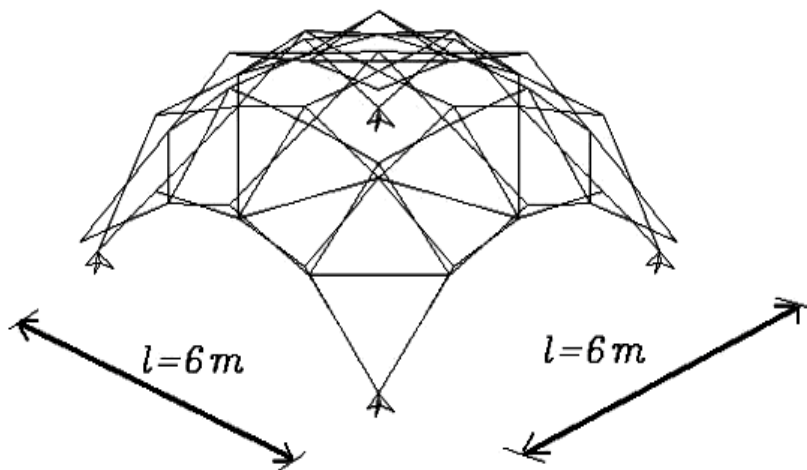


Figure 10. A 80-uniplet foldable dome

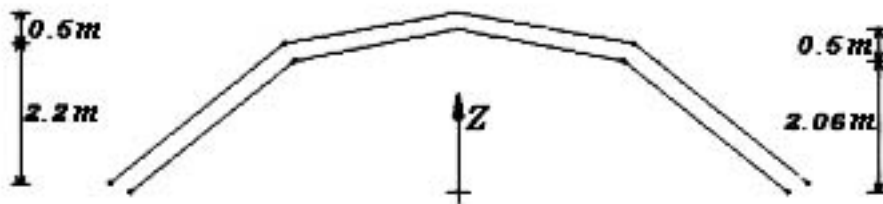


Figure 11. The height of nodes in diagonal view

Table 9: Numbering of nodes and member ordering

Group numbers	Design variables	Nodal numbers of the 3-node members	Member numbering
1	X_1	1-51-27,2-52-26,1-55-31,6-56-26 1-53-29,4-54-26,1-57-33,8-58-26	1,2,5,6 41,42,45,46
2	X_2	13-107-37,12-108-38,15-113-41,16-114-40 23-129-40,24-130-48,21-123-45,20-124-46 11-105-37,12-106-36,25-99-49,24-100-50 17-115-41,16-116-42,19-121-45,20-122-44	19,20,23,24 35,36,39,40 59,60,63,64 75,76,79,80
3	X_3	2-77-35,27-78-10,6-89-43,31-90-18 3-79-36,11-80-28,5-87-28,17-88-30 14-109-38,13-110-39,14-111-40,15-112-39 9-75-50,25-76-34,7-91-44,19-92-32 22-127-48,23-128-47,22-125-46,21-126-47 4-83-39,14-84-29,8-95-47,22-96-33 3-81-38,13-82-28,9-97-48,23-98-34 10-103-36,11-104-35,10-101-50,25-102-35 5-84-40,15-86-30,7-93-46,21-94-32 18-117-42,17-118-43,18-119-44,19-120-43	3,4,7,8 11,12,15,16 17,18,21,22 27,28,31,32 33,34,37,38 43,44,47,48 51,52,55,56 57,58,61,62 67,68,71,72 73,74,77,78
4	X_4	4-63-28,29-64-3,3-65-30,5-66-29 8-73-34,33-74-9,8-71-32,7-72-33 2-61-28,3-62-27,2-59-34,9-60-27 6-67-30,5-68-31,6-69-32,7-70-31	9,10,13,14 25,26,29,30 49,50,53,54 65,66,69,70

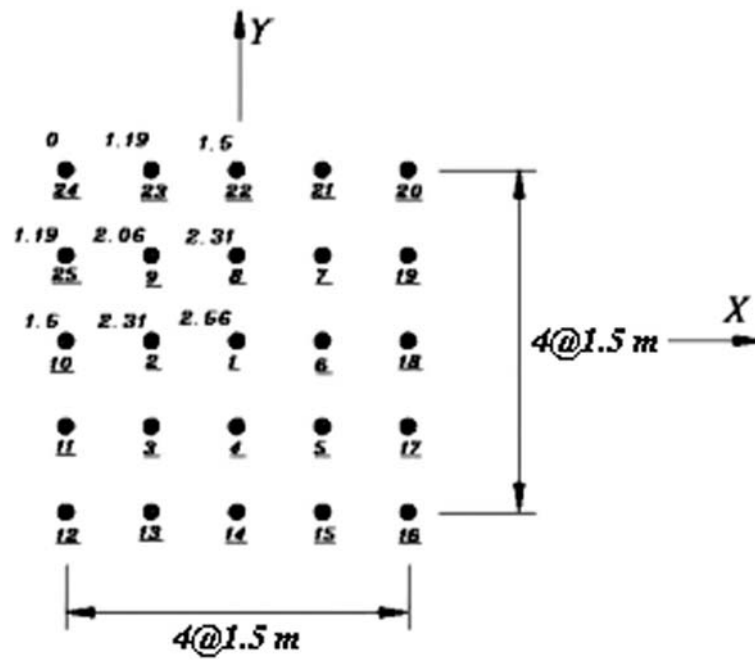


Figure 12. The dimensions of lower nodes

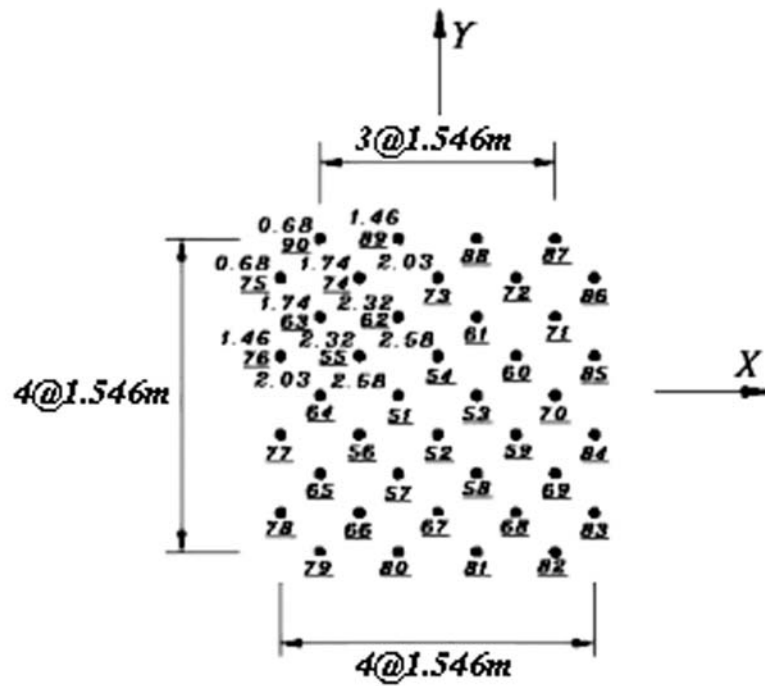


Figure 13. The dimensions of the middle nodes

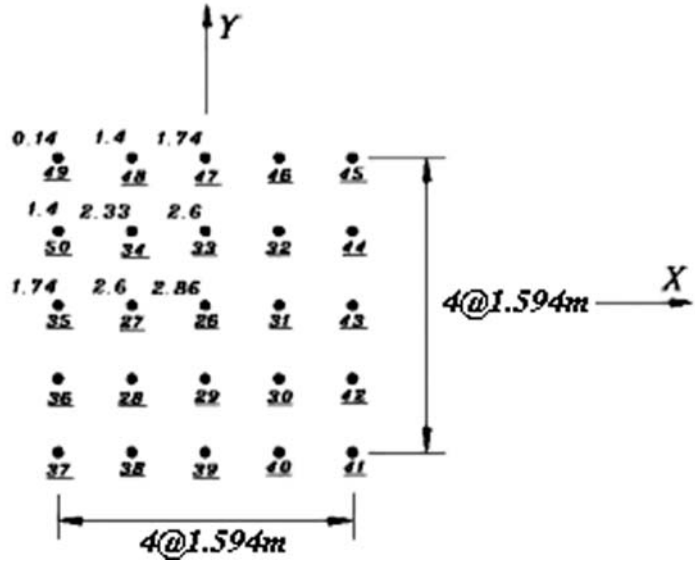


Figure 14. The dimensions of the upper nodes

Table 10: The discrete cross-section set

$S = \left\{ \begin{array}{l} 4^{cm} \times 4^{cm} \times I, 4^{cm} \times 3^{cm} \times I, 3^{cm} \times 3^{cm} \times I \\ 3^{cm} \times 2^{cm} \times I, 2^{cm} \times 2^{cm} \times I, 1^{cm} \times 1^{cm} \times 0.2^{cm} \end{array} \right\}, I = 0.2, 0.25, 0.3(mm)$

Table 11: Design data

Constraint data	Displacement constraints:	
	In all direction $ \Delta_j \leq 2cm (0.79 in)$	
Loading data	Stress constraints:	
	Relations 10,11,12,13,14	
Nodes	26,...,50	
	$P_z (kN)$	
Material properties	Modulus of elasticity $E = 2.06 \times 10^5 Mpa (3 \times 10^4 ksi)$	
	Weight density of the material $\rho = 7.4 \times 10^4 N/m^3 (0.27 lb/in^3)$	
	Yield stress $f_y = 2.35 \times 10^2 Mpa (34.08 ksi)$	

In Table 12 the results of optimal design are depicted. Figure 15 shows the history of optimization for generations. In this example, the number of generations is 50, the population size is 100, the mutation rate is 0.15, and the constant k_p for penalty function is taken 3.6.

Table 12: Results for the 80-uniplet foldable dome

Weight N (lb)	Design variables cm^2			
	X_1	X_2	X_3	X_4
2310.6 (519.2)	1.44	3.04	2.25	0.64
Sec. #	15	3	11	16

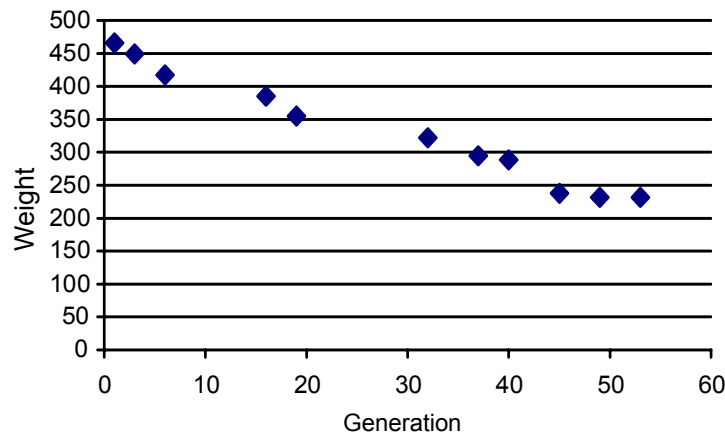


Figure 15 The history of optimization for the 80-uniplet foldable dome

5. CONCLUSIONS

The main emphasis of this paper is on the suitability of the genetic algorithm for the optimal design of foldable structures. Since the genetic algorithm does a probabilistic search in all discrete spaces and simulates random generations for an optimum point, it can be considered as a reasonable tool for optimization.

The present algorithm, achieves optimal designs with a good convergence. The selection method of elitism prevents the omitting of the best individuals. The high rate of mutation was considered to prevent local optimum.

Finally the use of uniplet, simplifies the analysis of foldable structures and hence increases the efficiency of optimization.

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